CCJ

微积分 上

CALCULUS - PART I

华南理工大学 微电子学院 微电子科学与工程 20 级 CCJ

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新的函数

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\sin^{-1} x$$

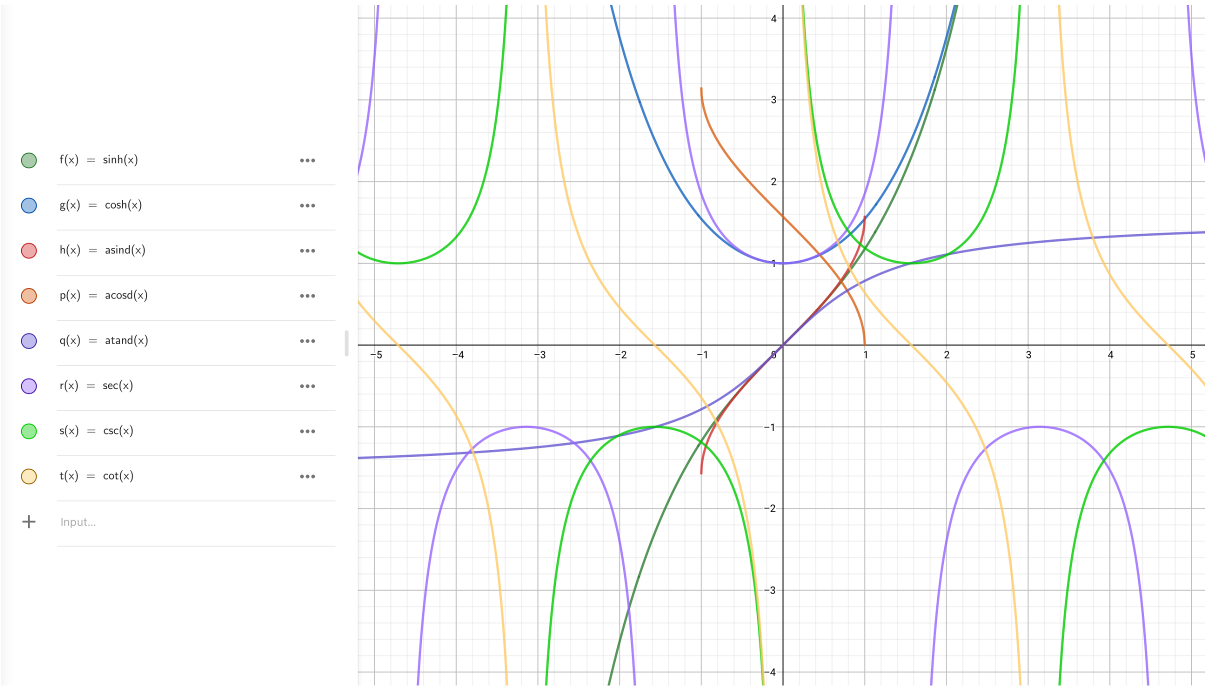
$$\cos^{-1} x$$

$$\tan^{-1} x$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{1}{\tan x}$$



常用极限

洛必达法则 L'Hospital's Rule

若$f(x), g(x)$满足极限同时为0/∞且连续可导，则

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}=A$$

其它结论

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$$

$$\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a$$

$$\lim_{n\to 0}(1+an)^{\frac{1}{n}}=e^a$$

$$\lim_{n\to 0}\frac{\ln(1+n)}{n}=1$$

$$\lim_{n\to 0}\frac{e^x-1}{n}=1$$

$$\lim_{n\to 0}\frac{\sin x}{x}=1$$

无穷小替换

$$\ln(x+1)\sim x$$

$$e^x+1\sim x$$

$$a^x+1\sim x\ln a$$

$$1-\cos x\sim\frac{1}{2}x^2$$

$$\sqrt[n]{1+x}\sim 1+\frac{x}{n}$$

$$\frac{1}{1+x}\sim 1-x$$

$$\sin x\ \sim x$$

$$\tan x\ \sim x$$

$$\arcsin x\ \sim x$$

$$\arctan x\ \sim x$$

求导结论

$$(x^r)' = rx^{r-1}$$

$$(|x|)' = \frac{|x|}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$(\cot x)' = -\frac{1}{\sin^2 x}$$

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$(\tanh x)' = \frac{1}{\cosh^2 x}$$

$$(\coth x)' = -\frac{1}{\sinh^2 x}$$

$$(\ln x)' = \frac{1}{x}$$

$$(\log_a x)' = \frac{1}{x \ln a}$$

$$(e^x)' = e^x$$

$$(a^x)' = a^x \ln a$$

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\tan^{-1} x)' = \frac{1}{1+x^2}$$

$$(\sec^{-1} x)' = -\frac{1}{|x|\sqrt{1-x^2}}$$

积分结论

分部积分 Integration by Parts

$$\int u\,dv = uv - \int v\,du + C$$

其它结论

$$\int \tan u\,du = -\ln|\cos u| + C\,, \qquad \int \cot u\,du = \ln|\sin u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C\,, \qquad \int \csc u\,du = \ln|\csc u - \cot u| + C$$

$$\int \frac{1}{\sqrt{a^2 - u^2}}\,du = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{a^2 + u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C\ , \qquad \int \frac{1}{a^2 - u^2}\,du = \frac{1}{2a}\ln\left(\frac{u+a}{u-a}\right) + C$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}}\,du = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C = \frac{1}{a}\cos^{-1}\left|\frac{a}{u}\right| + C \quad \left(\sec^{-1}x = \cos^{-1}\frac{1}{x}\right)$$

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C\,, \qquad \int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$\int \tan^2 u\,du = -u + \tan u + C\,, \qquad \int \cot^2 u\,du = -u - \cot u + C$$

$$\int \sin^3 u\,du = -\frac{1}{3}(2 + \sin^2 u)\cos u + C\,, \qquad \int \cos^3 u\,du = -\frac{1}{3}(2 + \cos^2 u)\sin u + C$$

$$\int_0^\infty u^n e^{-u}\,du = n!$$

$$\int_0^\infty e^{-au^2}\,du = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^{\frac{\pi}{2}} \sin^n u\,du = \int_0^{\frac{\pi}{2}} \cos^n u\,du = \begin{cases} \dfrac{1\cdot 3\cdot 5\cdot \ldots \cdot (n-1)}{2\cdot 4\cdot 6\cdot \ldots \cdot n}\cdot\dfrac{\pi}{2}, & n \textit{ is even.} \\ \dfrac{2\cdot 4\cdot 6\cdot \ldots \cdot (n-1)}{1\cdot 3\cdot 5\cdot \ldots \cdot n}, & n \textit{ is odd.} \end{cases}$$

三角函数诱导公式

$$\tan(s+t)=\frac{\tan s+\tan t}{1-\tan s\tan t}$$

$$\tan(s-t)=\frac{\tan s-\tan t}{1+\tan s\tan t}$$

$$\tan 2t=\frac{2\tan t}{1-\tan^2 t}$$

$$\sin\frac{t}{2}=\pm\sqrt{\frac{1-\cos t}{2}}$$

$$\cos\frac{t}{2}=\pm\sqrt{\frac{1+\cos t}{2}}$$

微分中值定理

费马引理 Fermat's Theorem

$$f'(x)_{min/max} = 0$$

罗尔中值定理 Rolle's Theorem

$$f(a) = f(b),\quad a \le c \le b,\ f'(c) = 0$$

拉格朗日中值定理 Lagrange's Theorem

$$f(a) - f(b) = f'(c)(a - b)$$

柯西中值定理 Cauchy's Theorem

$$\frac{f(a) - f(b)}{F(a) - F(b)} = \frac{f'(c)}{F'(c)}$$

泰勒展开 Taylor's Series

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)(x - x_0)^2}{2!} + \cdots + \frac{f^{(n)}(x_0)(x - x_0)^n}{n!} + 余项$$

$$f(x) - f(x_0) \approx f'(x_0)(x - x_0)$$

马克劳林公式 Maclaurin's Series

$$f(x) = f(0) + f'(0) \cdot x + \frac{f''(0) \cdot x^2}{2!} + \cdots + \frac{f^{(n)}(0) \cdot x^n}{n!} + 余项$$